PAPR Reduction of Complex Wavelet Packet Modulation (CWPM) System using a Novel Hybrid Technique over Selective Rayleigh Fading Channel

Hikmat N. Abdullah  
Al-Mustansiriyah University  
hikmat.abdullah@ieee.org

Majid A. Alwan  
Basrah University  
altimimee@yahoo.com

Fadhil S. Hasan  
Al-Mustansiriyah University  
fadel_sahib@yahoo.com

Abstract

High Peak to Average Power Ratio (PAPR) of transmitted signal is one of the major drawbacks of the complex wavelet packet modulation (CWPM). Utilizing the advantage of concentrating the energy to certain subspaces of the discrete wavelet transform, the performance of three methods to reduce PAPR in CWPM are investigated. These are: threshold method, clipping method and a proposed hybrid method combines the two previous ones (called TC method). The performance of the CWPM system has been measured by Bit Error Rate (BER) degradation, PAPR reduction and CCDF. The results show that the proposed hybrid technique gives higher PAPR reduction with less degradation in BER.

Keywords: Complex Wavelet Packet Modulation, Peak Average Power Ratio Reduction, Selective Rayleigh Fading Channel

1- Introduction

Complex Wavelet Packet Modulation (CWPM) is a multiplexing method where information carrying bits modulate a set of orthogonal wavelet packet waveforms which are then combined into a single composite signal. WPM is a promising alternative to the popular Fast Fourier Transform (FFT) based Orthogonal Frequency Division Multiplexing (OFDM). However, high peak to average power ratio (PAPR) that affects OFDM is also a problem in WPM. The large peaks increase the amount of inters modulation distortion resulting in an...
increase in the error rate. The average signal power must therefore be kept low to ensure that the transmitter amplifier operates in the linear region [1].

The survey of PAPR reduction in WPM can be summarized as follows: a multi-pass pruning method to reduce PAPR was proposed by Baro [2]. Zhang [3] suggested a threshold based method to reduce PAPR. Le et. al [4] derived upper bounds for the maximum PAPR for WPM transmission and based on these results wavelets that minimize PAPR are obtained. A novel adaptive companding transform scheme was proposed by Rostamzadeh to efficaciously reduce the PAPR of OFDM and WPDM signals [5]. Torun et. al [6] proposed a method to reduce the Peak-to-Average Power Ratio (PAPR) in the developmental Wavelet Packet Multi-carrier Modulation (WPM) system. A method works on the principle that the PAPR of a multicarrier system can be adjusted by varying the phase-shifts of the subcarriers. Hence different PAPR values for the same information can be obtained by randomly altering the phases of the sub-carriers used to modulate the data. The WPM frame with the least PAPR is then identified and transmitted.

In this paper, a hybrid technique composed of both the thresholding and clipping techniques is introduced to reduce PAPR in CWPM system. The rest of the paper is organized as fellows: section 2 presents the concept and structure of CWPM system. Section 3 describes the theory of threshold and clipping methods and introduces the proposed method. Section 4 shows the simulation results while the section 5 gives the important conclusions drawn through the work.

2- Complex Wavelet Packet Modulation (CWPM) System

Figure (1) illustrates a generic block diagram of CWPM transceiver. CWPM system employs two filter banks i.e. Inverse Discrete Wavelet Packet Transform (IDWPT) (reconstruction) placed at the transmitter side, and Discrete Wavelet Packet Transform (DWPT) (decomposition) placed at the receiver side. The block "MAKE CMPLX" accepts two N-dimensional real vectors as inputs. Its output is an N-dimensional complex vector whose ith complex element is formed from the ith real elements of the two input vectors. The input to the inverse wavelet transform is DQPSK complex symbols, therefore, there are two IDWPT blocks, one for real symbols and the other for imaginary symbols. The output of two IDWPT is combined together in complex form to introduce the transmitted signal \( x[n] \) in form:

\[
x[n] = \sum_{k=1}^{N} \sum_{j=0}^{N-1} a_{k,j} \varphi_j(n - k N) + i \sum_{j=0}^{N-1} b_{k,j} \varphi_j(n - k N)
\]

where \( a_{k,j} \), \( b_{k,j} \) are a real and imaginary constellation encoded kth data symbol modulating the jth wavelet packet basis function respectively.
The IDWPT synthesizes a discrete representation of the transmitted signal as the sum of $N$ waveforms shifted in time that embed information about data symbols. The DWPT at the receiver recovers the transmitted symbols $a_{k,j}$, $b_{k,j}$ through the analysis formula exploiting orthogonality properties of DWPT.

### 3- Peak Average Power Ratio (PAPR) Reduction

The PAPR of the base band transmitted signal $x(t)$ is defined as the ratio of maximum power of the transmitted signal over the average power. The PAPR of an OFDM signal in analog domain can be represented as [5]:

$$\text{PAPR} = \max_{0 \leq t \leq T_s} \left| x(t) \right|^2 \over E\left( \left| x(t) \right|^2 \right)$$  \hspace{1cm} (2)

Nonlinear distortion in HPA occurs in the analog domain, but the most of the signal processing operation for PAPR reduction occur in the digital domain. The PAPR of discrete time signal is given as [6]:

$$\text{PAPR} = \max_n \left( \left| x(n) \right|^2 \right) \over E\left( \left| x(n) \right|^2 \right)$$  \hspace{1cm} (3)

where $E(.)$ denotes ensemble average calculated over the duration of WPDM symbols. The Complementary Cumulative Distribution Function (CCDF) of the PAPR is one of the most frequently used performance measures for PAPR reduction techniques. The CCDF of the PAPR denotes the probability that the PAPR of data block exceeds a given certain value, and is expressed as follows [2]:

![Figure (1) Block diagram of the WPMCM system model. Arrows with a strike indicate complex quantities.](image-url)
CCDF (PAPR0) = Pr {PAPR > PAPR0}  \hspace{1cm} (4)

From Central limit theorem it follows that for a large value of subcarriers N, the real and imaginary component of the multicarrier signal are modeled as a zero mean Gaussian distribution random variable with variance \( \sigma^2 \). The amplitude of the OFDM signal therefore has a Rayleigh distribution and its power distribution becomes a central chi-square distribution with two degrees of freedom and zero mean [6]. The CCDF of the PAPR can be calculated as:

\[
Pr(PAPR \leq PAPR0) = 1 - \left(1 - e^{-PAPR0}\right)^N
\]

The distribution obtained by the conventional analysis, however, does not fit those of the PAPR of the OFDM signals obtained by computer simulations, even for very large N. In [8], Van Nee and Prasad gave an empirical approximation:

\[
CCDF(PAPR0) = 1 - \left(1 - e^{-PAPR0}\right)\alpha^N
\]

where \( \alpha \) is a parameter determined by computer simulation to be 2.8 [6].

### 3.1 The Threshold Method

The discrete wavelet transform (DWT) is a type of batch processing, which analyses a finite length time domain signal by breaking up the initial domain in two parts: the details and the approximated information. The approximation domain is successively decomposed into detail and approximation domains. We use the properties of the discrete wavelet transform that the DWT is scattered. This means only few coefficients of DWT dominates the representation. This property is widely used in image processing, such as wavelet de-noising. Using this properly in WPM systems, we can reduce the PAPR with little reconstruction loss [3]. Figure (2) shows the composite threshold method to the transmitter of a CWPM system.

Let \( x(n) \) be the signal obtained after orthogonal modulation. Since wavelet transforms always concentrate energy on some given number of bases, we can introduce a threshold \( T \) and compare it with the energy of each orthogonal base.

The standard thresholding of wavelet coefficients is governed mainly by either 'hard' or 'soft' thresholding function. In hard thresholding, the wavelet coefficients (at each level) below threshold \( T \) are made zero and coefficients above threshold are not changed whereas in soft thresholding, the wavelet coefficients are shrunk towards zero by an offset \( T \). The hard thresholding function is given as [8]:

\[
\text{Hard Thresholding: } w_i = \begin{cases} 
0, & |w_i| < T \\
\text{sgn}(w_i)w_i, & |w_i| \geq T 
\end{cases}
\]

\[
\text{Soft Thresholding: } w_i = \begin{cases} 
0, & |w_i| < T \\
|w_i| - T, & |w_i| \geq T 
\end{cases}
\]
Figure (2) Block diagram to show the threshold of a

$$x_T(n) = \begin{cases} 0 & \text{if } |x(n)| < T \\ x(n) & \text{if } |x(n)| \geq T \end{cases}$$  
(7)

where $x_T(n)$ represents the output value after thresholding and $T$ is threshold value. Donoho et al. [8] introduced various shrinkage rules based on different threshold values and thresholding functions such as "visushrink" with fixed universal threshold:

$$T = \sigma \sqrt{2 \log N}$$  
(8)

with  
$$\sigma = \frac{\text{MAD}(x)}{0.6745}$$  
(9)

where MAD($x$) is median absolute deviation of $x(n)$ signal, and $N$ is the subcarrier number. Therefore, we can use this threshold as proposed threshold to reduce the PAPR in CWPM system.

Assume that there are $M$ basis functions whose energies are smaller than the threshold $T$. Then, let us define a new sequence $x_1(n)$:

$$x_1(i) = x_T(n), \quad x_T(n) \neq 0, \quad i = 0,1,...,N-M-1; \quad n = 0,1,...,N-1 \quad \text{(10)}$$

The PAPR can now be written as:

$$\text{PAPR}_N = 10 \log_{10} \left( \max \left\{ \left| x_1(n) \right|^2, \text{for } n = 0,1,...,N-M-1 \right\} \right)$$  
(11)

Two measures are used, the PAPR reduction and BER degradation measures, to test the performance of threshold method as well as the CCDF and BER measures. The mean PAPR reduction (in dB) is obtained by:

$$\text{PAPRR} = \text{PAPR} - \text{PAPR}_T$$  
(12)
where PAPR represents the PAPR without threshold, and PAPR$_T$ represents the PAPR after threshold is taken.
And the BER degradation is obtained by:

$$\text{BERD} = \text{BER} - \text{BER}_T$$  \hspace{1cm} (13)

where BER represents the bit error rate without threshold, and BER$_T$ represents the bit error rate after threshold is taken.

3.2 The Clipping method

The clipped MCM signal is presented as [10]:

$$x_c(n) = \begin{cases} 
x(n) & \text{if} |x(n)| \leq A_{\max} \\
A_{\max}e^{j\theta(n)} & \text{if} |x(n)| > A_{\max} 
\end{cases}$$  \hspace{1cm} (14)

where $\theta(n) = \arg[x(n)]$. Therefore the magnitude of the clipped signal does not exceed the threshold $A_{\max}$, and the phase of $x(n)$ is preserved. The clipping severity is measured by the clipping ratio, defined as [10]:

$$\text{CR} = \frac{A_{\max}}{\sqrt{P_s}}$$  \hspace{1cm} (15)

where $P_s$ is the average power of $x(n)$ and it is given by:

$$P_s = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$  \hspace{1cm} (16)

The clipping causes both in-band and out-of-band distortion because of nonlinear operation of the clipping, the in-band distortion causes degradation of BER, while the distortion also caused out-of-band emission.

3.3 The proposed Hybrid Method

By combing the concepts of both the threshold and clipping techniques, a proposed formula can be used to reduce the PAPR of CWPM signal as follows:

$$y(n) = \begin{cases} 
0 & \text{if} |x(n)| < T \\
x(n) & \text{else if} |x(n)| \leq A_{\max} \\
A_{\max}e^{j\theta(n)} & \text{else if} |x(n)| > A_{\max} 
\end{cases}$$  \hspace{1cm} (17)
The above formula is simply a composition of equations 7 and 16. The evaluation of the performance of the proposed method as compared with traditional ones for the same values of $T$ and $A_{\text{max}}$ will considered in the next section.

4- Simulation Results

The simulation results of CWPM systems are obtained by using MATLAB version 7.12 (R2011a). The PAPR reduction methods mentioned in section 3 are compared over the frequency selective Rayleigh fading channel. Table (1) shows the parameters of the implemented CWPM system.

<table>
<thead>
<tr>
<th>Table (1) Simulation parameters</th>
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</thead>
<tbody>
<tr>
<td><strong>Modulation Type</strong></td>
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<tr>
<td><strong>Number of subcarriers</strong></td>
</tr>
<tr>
<td><strong>Doppler spread factor (fdTs)</strong></td>
</tr>
<tr>
<td><strong>Path delay</strong></td>
</tr>
<tr>
<td><strong>Path gain</strong></td>
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<tr>
<td><strong>Channel model</strong></td>
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</tbody>
</table>

Figure (3) shows the curves of mean and maximum PAPRR in terms of threshold $T$. Figure (4) shows the BER degradation in terms of threshold $T$. From these figures, it can be seen that when $T$ is increased, $M$ is increased and hence lower PAPR is obtained but BER performance is decreased, this is due to that if $T$ increased more information is discarded from the transmitted signal and hence the signal is distorted. Therefore there is tradeoff between PAPR and BER performance. Types of wavelet family (Daubechies, Coiflet, or Symlet) perform different in reducing PAPR. Symlet appears to be the higher reduction in PAPR with the least degradation in BER. Increase the length of filter will increase PAPR reduction but this will increase the degradation in BER.

Figure (5) shows CCDF of the PAPR for different values of $T$. Figure (6) shows BER performance for the case of with and without threshold for different values of $T$. Figure (7) shows the CCDF of the PAPR of the CWPM signal for different types of wavelet family. Figure (8) shows BER for different wavelet family, with and without threshold method using Donoho Threshold. From these figures, it is seen that increased $T$ will be decreased PAPR but increased BER degradation and the proposed threshold gives PAPR reduction of about 2 dB with a very small degradation in BER (less than 0.5 dB).

Figure (9) shows the curves of the mean PAPRR in terms of the clipping ratio $\gamma$. Figure (10) shows the BER degradation in terms of the clipping ratio $\gamma$. Figure (11) shows CCDF of the PAPR for different values of the clipping ratio (CR) for $N=128$ and db6. Figure (12) shows BER performance for the case of with and without clipping for different values of the clipping ratio (CR) for $N=128$ and db6. From these figures, it is seen that Symlet family gives
higher PAPR reduction but this effect on BER. Also it can be seen that increasing the length of Daubechies filter has a slight effect on PAPR reduction. When clipping ratio (CR) about 4 dB, PAPR reduction is about 4 dB at CCDF=0.01 with BER degradation less than 0.5 dB.

Figure (3) The mean and maximum reduction of PAPR as a function of threshold T

Figure (4) The BER degradation as a function of threshold T.
Figure (5) CCDF of the PAPR of the CWPM signal for different values of threshold $T$

Figure (6) BER performance for the case of with and without threshold for different values of $T$
Figure (7) CCDF of the PAPR of the CWPM signal for different types of wavelet family

Figure (8) BER performance comparison for different wavelet family with and without threshold
Figure (9) The mean reduction of PAPR as a function of Clipping Ratio (CR)

Figure (10) The BER degradation as a function of Clipping Ratio (CR)
Figure (11) The CCDF of clipped signal’s PAPR for various value of Clipping Ratio (CR)

Figure (12) the BER degradation in terms of the clipping ratio.

Figure (13) shows the CCDF of the PAPR for the proposed method for various values of clipping ratio (CR) where Donoho threshold is used as T threshold. Figure (14) shows BER performance of proposed method for various values of clipping ratio (CR) with Donoho threshold too. From these figures, it is seen that for CR=4 dB, PAPR reduction about 7 dB at CCDF=0.01 with BER degradation less than 0.5 dB.
Figure (13) The CCDF of the PAPR for TC method for various values of Clipping Ratio (CR)

Figure (14) BER performance of clipped signal for various value of Clipping Ratio (CR)

5- Conclusions

In this paper the PAPR reduction of CWPM using novel hybrid technique that combines threshold and clipping method is investigated. The results show that, Donoho threshold gives good estimation and more sensitive for the energy of transmitted signal and can be used as proposed threshold to reduced PAPR in CWPM. The threshold method gives PAPR reduction 2-3dB with BER degradation less than 0.5dB when using Donoho threshold. Clipping method is another distortion method can be used to reduce PAPR in multicarrier modulation system.
but this method effect the performance of BER when increasing the level of clipping. For CR= 4dB, it is found that the PAPR reduction is about 4dB while BER degradation less than 0.5dB. The proposed TC method combined both threshold and clipping method and gives PAPR reduction about 7dB for CR=4dB with BER degradation less than 0.5dB. Therefore the proposed method gives PAPR gain about 4dB greater than threshold method alone and about 3 dB greater than clipping threshold alone.

References


